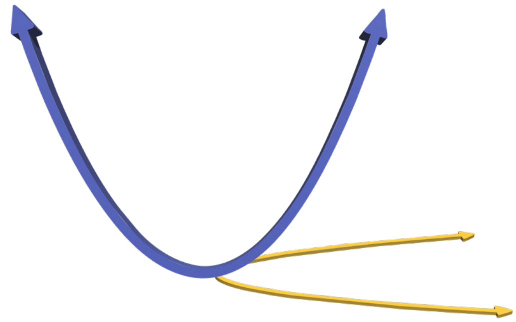
Unit 1 TRANSFORMATIONS OF FUNCTIONS



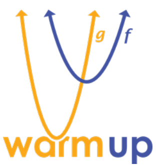
1.1 Prerequisite Skills + Translations of Functions

Part A – Prerequisite Skills

Review of Two Familiar Functions

In this units we'll take some known and new functions and apply various transformations. And that means, if you're eager with anticipation, to alter the function's equation or graph.

However before we get into all of that – over the next few pages (and 6 warm-ups), we'll brush up on some key concepts we'll need in this first unit and throughout this course. Starting with – some functions from Math 20!



Warm-up **Exploration #1**

The Quadratic Function – The Graph of $y = x^2$

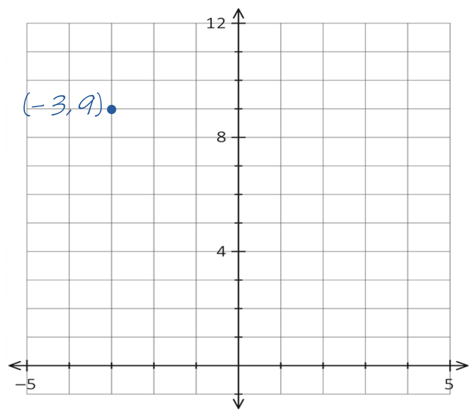
1 ➡ **Complete** the **table of values** on the right, and **plot** the points to **sketch** the graph.

2 ➡ **State** the **domain** and **range** of the function.

Domain ________ Range ________

3 ➡ On the same grid, **sketch the graph** of $y = x^2 + 3$. *Add 3 to all y-coordinates, verify on your graphing calc.*

x	$y = x^2$
-3	$(-3)^2 = 9$
-2	
-1	
0	
1	
2	
3	



Visit math30-1power.com for video tutorials of all lessons including warm-up explorations.

The Absolute Value Function – the Graph of $y = |x|$

4 ➡ **Complete** the **table of values** on the right, and **plot** the points to **sketch** the graph.

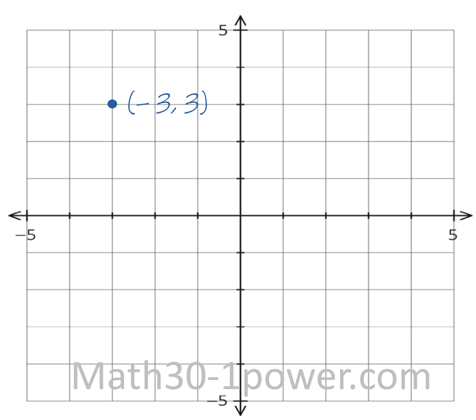
5 ➡ **State** the **domain** and **range** of the function.

Domain ________ Range ________

6 ➡ On the same grid, **sketch the graph** of $y = |x| - 2$.

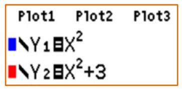
Explain how the graph compares to $y = |x|$.

x	$y = \|x\|$
-3	$\|-3\| = 3$
-2	
-1	
0	
1	
2	
3	

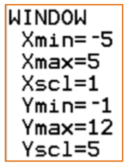


CALC TIP

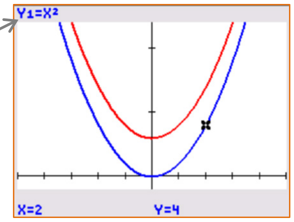
Try yourself – Confirm the graphs on your calculator.



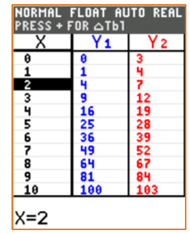
Match the window to the given grid



Toggle between the two graphs using the arrow buttons



Confirm points using: trace

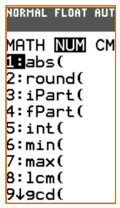


Also confirm points using **table** (2nd graph)

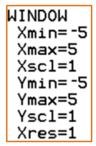
To graph $y = |x|$, you'll need to find the *abs(* function.

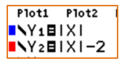
math ▸

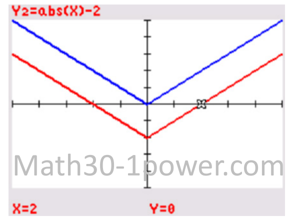
Key in "MATH" then choose the first option from the "NUM" menu



Match the window again....







Try changing the constant term, how does that change things?

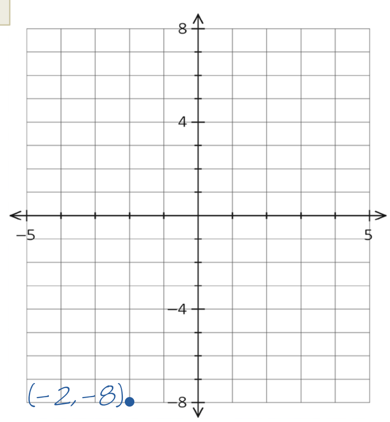
Preview of a Few New Functions

Over the next two warm-ups, we'll preview three functions we'll see much of later in the course.

Warm-up
Exploration #2

The Cubic Function – The Graph of $y = x^3$

x	$y = x^3$
-2	$(-2)^3 = -8$
-1	
0	
1	
2	



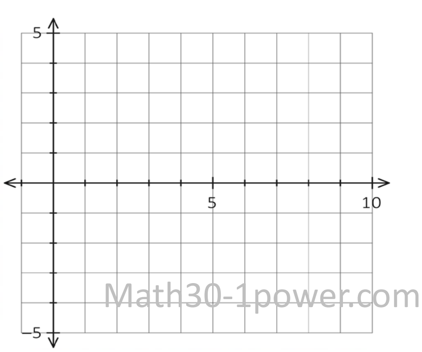
1 ➡ **Complete** the **table of values** on the left, and **plot** the points to **sketch** the graph of the function. *Use your graphing calculator to confirm.*

2 ➡ **State** the function's **domain** and **range.**

Domain ________ Range ________

The Radical Function – The Graph of $y = \sqrt{x}$

x	$y = \sqrt{x}$
0	$\sqrt{0} = 0$
1	
4	
9	
-1	



3 ➡ **Complete** the **table of values** on the left, and **plot** the points to **sketch** the graph of the function. *Use your graphing calculator to confirm.*

4 ➡ **State** the function's **domain** and **range.**

Domain ________ Range ________

5 ➡ **Compare** this table of values with that of $y = x^2$. What do you notice?

6 ➡ On the same grid, **sketch** the graph of $y = \sqrt{x} - 4$. **State** the **domain** and **range** of this new function.

Domain ________ Range ________

Exploration #3 The Rational Function – The Graph of $y = \frac{1}{x}$

1 ➡ **Complete** the (partial) **table of values** on the left. *The points are already plotted on the graph.*

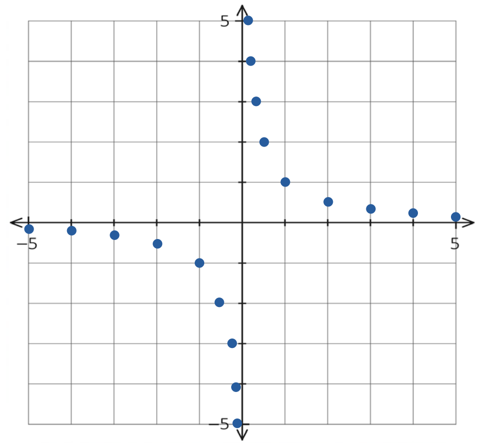
2 ➡ **Graph** $y = 1/x$ on your graphing calculator, using the window shown.

x: $[-5, 5, 1]$ y: $[-5, 5, 1]$

min max scl

3 ➡ With the help of you calculator, **sketch** the graph by connecting the plotted points in a smooth curve.

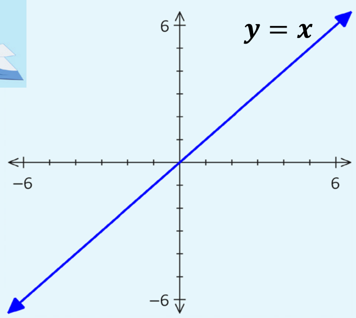
x	$y = 1/x$
-5	1/−5 = −0.2
-4	1/−4 = −0.25
-1	
0	
0.2	
0.25	
⋮	⋮



4 ➡ *Fill in the blanks:* The graph of $y = \frac{1}{x}$ has a vertical asymptote at _______ and a horizontal asymptote at _______. The domain of the function is ___________ and the range is ___________.

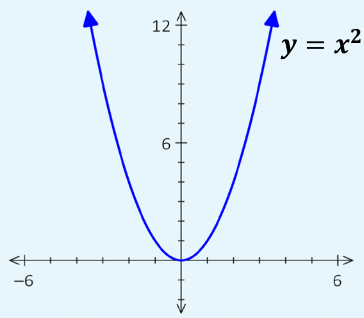
Can't divide by zero

Some of the **basic functions** graphs we should be familiar with are:



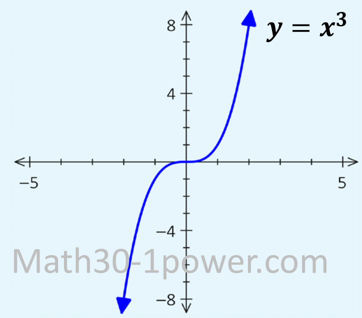
Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R}\}$



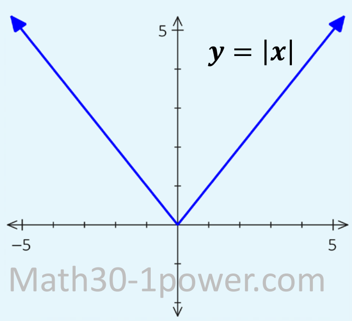
Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \geq 0, y \in \mathbb{R}\}$



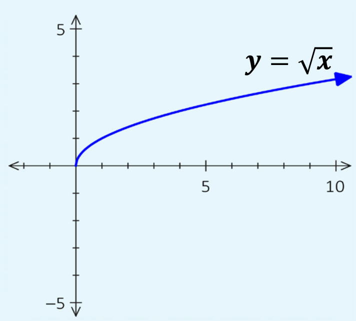
Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R}\}$



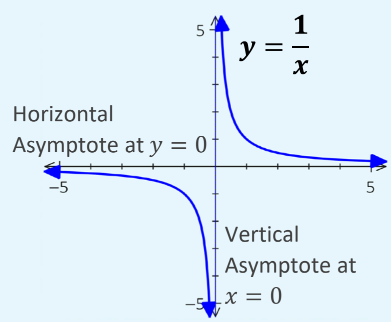
Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \geq 0, y \in \mathbb{R}\}$



Domain: $\{x \geq 0, x \in \mathbb{R}\}$

Range: $\{y \geq 0, y \in \mathbb{R}\}$



Domain: $\{x \neq 0, x \in \mathbb{R}\}$

Range: $\{y \neq 0, y \in \mathbb{R}\}$

Interval Notation of Domain and Range

You are likely familiar with the formats above, *set notation*. In this course we also use *interval notation*:

So, Domain: $\{x \in \mathbb{R}\}$ can be written in interval notation: $(-\infty, \infty)$ Read as: *"from $-\infty$ to ∞"*

Rounded brackets, do not include endpoints

And, Range: $\{y > 0, y \in \mathbb{R}\}$ can be written: $[0, \infty)$ Read as: *"from 0 to ∞, including 0"*

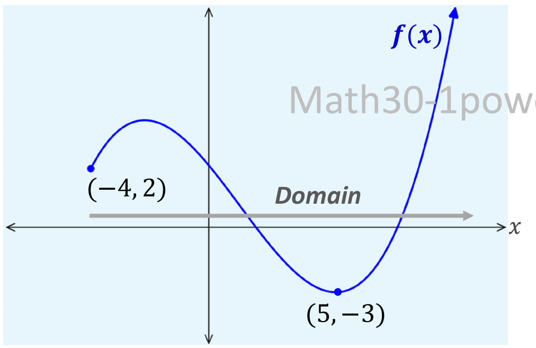
Square bracket, endpoint is included

Exploration #4 Domain and Range Fundamentals

The concept of **domain** and **range** is highly important in this course. In this next warm-up, we'll look at how to determine the domain and range from a **graph**, and how to determine the domain from the **equation** of a **function**.

Determining Domain and Range from the graph of a function

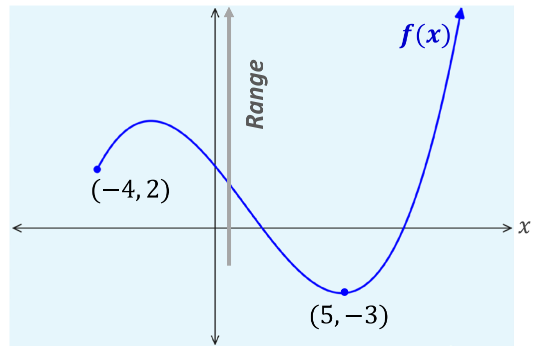
For **domain**, we consider all input, or $\boldsymbol{x}$ values.



Domain is: $\{x \geq -4, x \in \mathbb{R}\}$ *Set notation*

or, alternatively: $[-4, \infty)$ *Interval notation*

For **range**, we consider all output, or $\boldsymbol{y}$ values.



Range is: $\{y \geq -3, y \in \mathbb{R}\}$ *Set notation*

or, alternatively: $[-3, \infty)$ *Interval notation*

Determining Domain from the equation of a function

Given the equation of a function, we need to exclude any non-permissible values. (That is, we need to state any restrictions)

Which is actually easier than might sound! Because in this course, there are only three restrictions we need to consider. Ready? Just remember that we can't:

1 Divide by Zero *For example, what is the domain of....*

$$f(x) = \frac{5x+1}{x-3}$$

We do not need to graph on our calculators, or even consider what the graph might look like. Simply think .... For what value(s) x would the denominator ("bottom") be zero?

Domain is given by: $x - 3 \neq 0$ Set the denominator *not equal to zero*, and isolate x

$\{x \neq 3, x \in \mathbb{R}\}$ This domain is not suitable for interval notation, but if we chose to, it would be: $(-\infty, -3) \cup (-3, \infty)$

"**Union**" (think – "*combined with*")

Note that this is not in the curriculum

2 Square Root Negatives *For example, what is the domain of....*

$$g(x) = \sqrt{4x+3}$$

Again, we need not concern ourselves with the graph! (And like $f(x)$ above, we won't even get to the graphs until unit 7) Instead, think .... For what value(s) x would we be square-rooting negatives?

Domain is given by: $4x + 3 \geq 0$ Set what's under the root sign greater than or equal to zero, and isolate x

$4x \geq -3$

$\{x > -3/4, x \in \mathbb{R}\}$ *In interval notation:* $[-3/4, \infty)$

3 Take the Logarithm* of 0 or Negatives

*Let's pin this for now – we'll come back in unit 3!

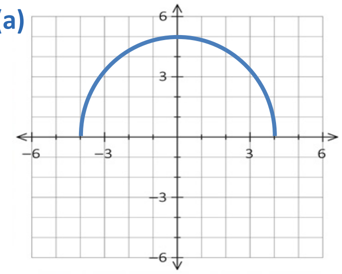
Before we continue our warm-ups and into transformations, let's do some practice with **domain** and **range**.

Class Example 1.11 *Obtaining Domain and Range from a Graph*

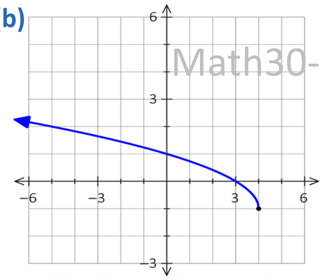
Answers are on bottom of the next page

Given the graphs below, state the domain (D) and range (R) for each function, in both set and interval notation

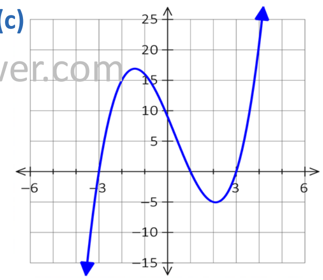
(a)



(b)



(c)



(a)
D: ______________ ______________
Set Notation *Interval Notation*

R: ______________ ______________
Set Notation *Interval Notation*

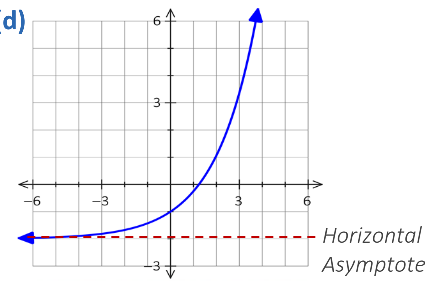
(b)
D: ______________ ______________
Set *Interval*

R: ______________ ______________
Set *Interval*

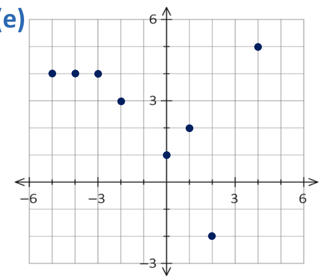
(c)
D: ______________ ______________
Set *Interval*

R: ______________ ______________
Set *Interval*

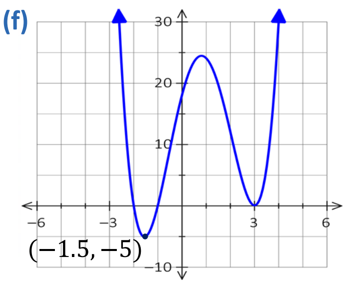
(d)



(e)



(f)



(d)
D: ______________ ______________
Set *Interval*

R: ______________ ______________
Set *Interval*

(e)
D: ______________
Set

R: ______________
Set

(f)
D: ______________ ______________
Set *Interval*

R: ______________ ______________
Set *Interval*

Class Example 1.12 *Obtaining Domain from a function equation*

Answers are on the bottom of the next page

Without graphing, determine the domain of each function below.

(a) $f(x) = \sqrt{5 - 3x}$ Provide in Set *and* Interval notation

(b) $p(x) = 3x^4 - x^3 + 2x - 1$ Set *and* Interval notation

(c) $y = -|x + 4| + 1$

(d) $g(x) = \dfrac{x + 1}{5x + 7}$

(e) $f(x) = \dfrac{1}{x^2 - 9}$

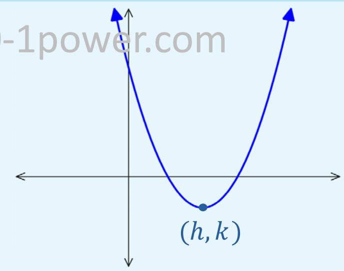
(f) $f(x) = \dfrac{1}{x^2 + 1}$

Exploration #5 Vertex Form of a Quadratic Function – The Graph of $y = a(x-h)^2 + k$

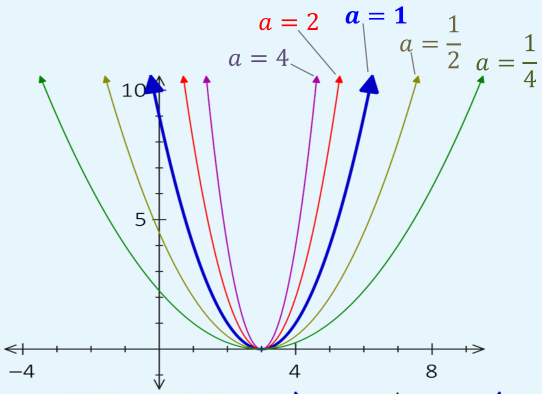
In Math 20, we saw how a **quadratic function** could be expressed $y = a(x-p)^2 + q$, where the coordinates of the vertex are (p, q).

In Math 30, the vertex form equation is $y = a(x-h)^2 + k$

Where the coordinates of the vertex are (h, k).



We also saw how while h and k affect the **position** of the graph, a affects the **shape** (how wide / narrow) and the **orientation**. (opens up / down)



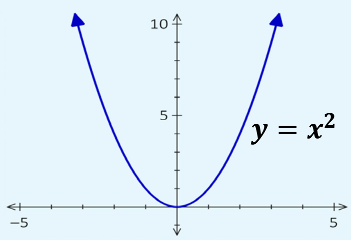
For example, consider the graph of $y = a(x-3)^2$, for different values of a.

The center function is where $a = 1$, that is, $y = (x-3)^2$

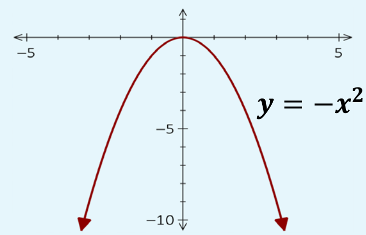
When $a > 1$, such as with $y = 2(x-3)^2$, the graph is *vertically stretched by a factor of 2*. (The graph is narrower)

And when $0 < a < 1$, such as with $y = \frac{1}{4}(x-3)^2$, the graph is *vertically stretched by a factor of 1/4*. (graph is wider)

Finally, recall that when $a > 0$, the graph opens **up**.



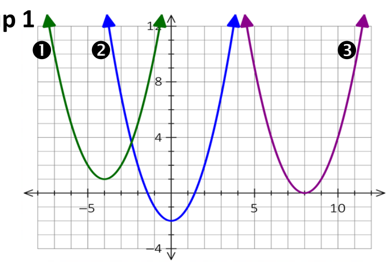
And when $a < 0$, the graph opens **down**.



1 Each of the graphs in group 1 represent a quadratic function in the form $y = a(x-h)^2 + k$, where $a = 1$ and $h, k \in I$
Determine an equation for each graph.

❶ ________________ ❷ ________________ ❸ ________________

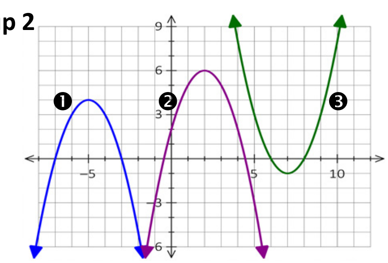
Group 1



2 Each of the graphs in group 1 represent a quadratic function in the form $y = a(x-h)^2 + k$, where $a = 1$ *or* $a = -1$, and $h, k \in I$
Determine an equation for each graph.

❶ ________________ ❷ ________________ ❸ ________________

Group 2



Answers from previous page

1.11 (a) D: $\{-4 \le x \le 4, x \in \mathbb{R}\}$ *or* $[-4, 4]$
R: $\{0 \le y \le 5, y \in \mathbb{R}\}$ *or* $[0, 5]$

(b) D: $\{x \le 4, x \in \mathbb{R}\}$ *or* $(-\infty, 4]$
R: $\{y \ge -1, y \in \mathbb{R}\}$ *or* $[-1, \infty)$

(c) D: $\{x \in \mathbb{R}\}$ *or* $(-\infty, \infty)$
R: $\{y \in \mathbb{R}\}$ *or* $(-\infty, \infty)$

(d) D: $\{x \in \mathbb{R}\}$ *or* $(-\infty, \infty)$
R: $\{y > -2, y \in \mathbb{R}\}$ *or* $(-2, \infty)$

(e) D: $\{-5, -4, -3, -2, 0, 1, 2, 4\}$
R: $\{-2, 1, 2, 3, 4, 5\}$

(f) D: $\{x \in \mathbb{R}\}$ *or* $(-\infty, \infty)$
R: $\{y \ge -5, y \in \mathbb{R}\}$ *or* $[-5, \infty)$

1.12 (a) $5 - 3x \ge 0$ → $5 \ge 3x$ → $x \le 5/3$ D: $\{x \le 5/3, x \in \mathbb{R}\}$ or $(-\infty, 5/3]$

(b) D: $\{x \in \mathbb{R}\}$

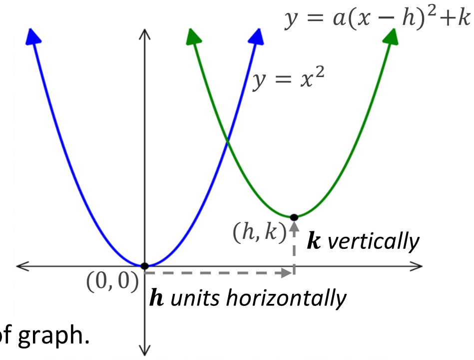
(c D: $\{x \in \mathbb{R}\}$

(d) $5x + 7 \ne 0$ → $5x \ne -7$ → $x \ne -7/5$
D: $\{x \ne -7/5, x \in \mathbb{R}\}$

(e) $x^2 - 9 \ne 0$
$(x+3)(x-3) \ne 0$ → $x \ne \pm 3$
D: $\{x \ne \pm 3, x \in \mathbb{R}\}$

(f) $x^2 + 1 \ne 0$ → $x^2 \ne -1$
D: $\{x \in \mathbb{R}\}$
Note: *$x^2 + 1$ is always positive (irreducible)*

Part B – Horizontal and Vertical Translations



On the previous page, we saw how the parameters a, h, and k affected the graph of $\boldsymbol{y = a(x-h)^2+k}$.

We can think of the vertex as having shifted, or **translated**, from:

- $(0, 0)$ on the graph of $y = x^2$ to (h, k)
- (h, k) on the graph of $y = a(x-h)^2+k$

A **transformation** of a function alters the location, shape or orientation of graph.

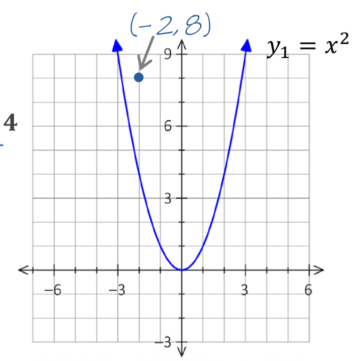
A horizontal or vertical **translation** is a “shift”, or change to the graph position. (Think of picking up and moving a graph left / right and up / down)

Exploration #6 Exploring the Effect of $\boldsymbol{h}$, $\boldsymbol{k}$ in $\boldsymbol{y = f(x-h)+k}$

1 ➡ Complete each table of values below and plot the points to **sketch** the second function, y_2, on the same grid as $y_1 = x^2$. Verify your graph of y_2 using your graphing calculator. (Match your window to the grid below)

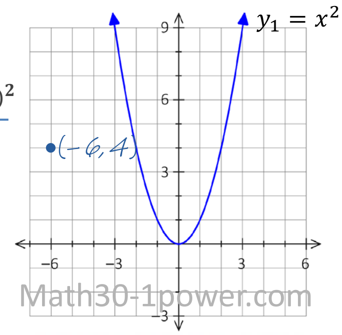
i $y_1 = x^2$
$y_2 = x^2 + 4$

x	$y = x^2$	$y_2 = x^2 + 4$
-2	$(-2)^2 = 4$	$4+4=8$
-1		
0		
1		
2		



ii $y_1 = x^2$
$y_2 = (x+4)^2$

x	$x + 4$	$y=(x+4)^2$
-6	$-6+4=-2$	$(-2)^2=4$
-5		
-4		
-3		
-2		



2 ➡ For each case, **describe** how the graph of $\boldsymbol{y_2}$ can be obtained by horizontally or vertically translating the graph of $\boldsymbol{y_1}$.

i $y_1 = x^2$
$y_2 = x^2 + 4$

ii $y_1 = x^2$
$y_2 = (x+4)^2$

3 ➡ For each case above, describe which coordinate (x or y) is affected, and how. Complete a mapping rule for each.

i $(x, y) \rightarrow$

ii $(x, y) \rightarrow$

4 ➡ Graph each of the following pairs of functions in your graphing calculator. Then, describe how the graph of $\boldsymbol{y_2}$ can be obtained by horizontally or vertically translating the graph of $\boldsymbol{y_1}$, and provide a **mapping rule**.

i $y_1 = \sqrt{x}$
$y_2 = \sqrt{x} - 3$

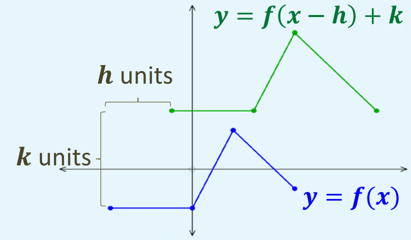
ii $y_1 = \sqrt{x}$
$y_2 = \sqrt{x-5}$

iii $y_1 = \sqrt{x}$
$y_2 = \sqrt{x+3} + 1$

A function $\boldsymbol{y = f(x)}$, transformed to $\boldsymbol{y = f(x-h)+k}$, is

- **Horizontally** translated $\boldsymbol{h}$ units: RIGHT if $h > 0$ LEFT if $h < 0$
- **Vertically** translated $\boldsymbol{k}$ units: UP if $k > 0$ DOWN if $k < 0$

A **mapping rule** describes the effect on each point on the original function to the transformed function. Here, it's: $(x, y) \rightarrow (x+h, y+k)$

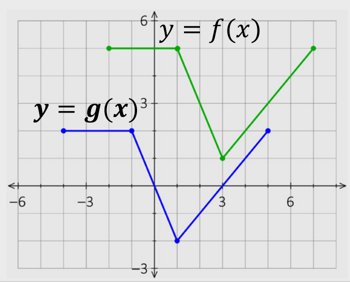


Note that the direction of the horizontal translation is the opposite of the sign

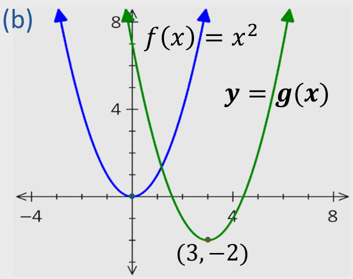
Worked Example

For each pair of functions below, $y = g(x)$ is obtained by horizontally and vertically translating the graph of $y = f(x)$. Determine an equation for $y = g(x)$, **(i)** in terms of $f(x)$ and **(ii)** in terms of x, for (b) and (c) only. Then, **(iii)** provide a mapping rule from $f(x) \rightarrow g(x)$.

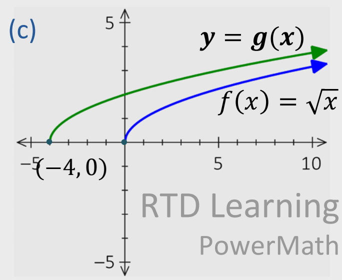
(a)



(b)



(c)



Solution: (a) $y = f(x)$ horizontally translates 2 units left, then vertically translates 3 units down

$\boldsymbol{g(x) = f(x+2) - 3}$

i - Equation in terms of f

$\boldsymbol{(x, y) \rightarrow (x-2, y-3)}$

ii - Mapping Rule

(b) $\boldsymbol{g(x) = f(x-3) - 2}$

i - Equation in terms of f

$\boldsymbol{g(x) = (x-3)^2 - 2}$

ii - Equation in terms of x

$\boldsymbol{(x, y) \rightarrow (x+3, y-2)}$

ii - Mapping Rule

(c) $\boldsymbol{g(x) = f(x+4)}$

i - Equation in terms of f

$\boldsymbol{g(x) = \sqrt{x+4}}$

ii - Equation in terms of x

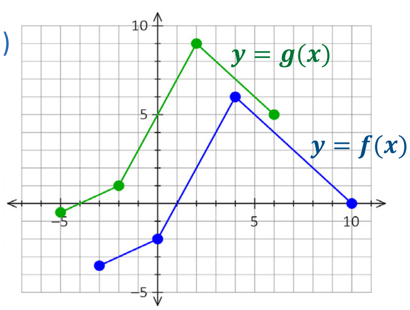
$\boldsymbol{(x, y) \rightarrow (x-4, y)}$

ii - Mapping Rule

Class Example 1.13 *Determining the Horizontal / Vertical Translation from a graph*

For each pair of functions below, $y = g(x)$ is obtained by horizontally and / or vertically translating the graph of $y = f(x)$. Provide the indicated equations / mapping rule below.

(a)



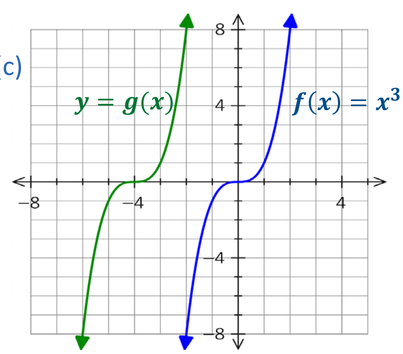
i ________________

Equation of $g(x)$ in terms of $f(x)$

ii ________________

Mapping rule of $y = f(x) \rightarrow y = g(x)$

(c)



i ________________

Equation of $g(x)$ in terms of $f(x)$

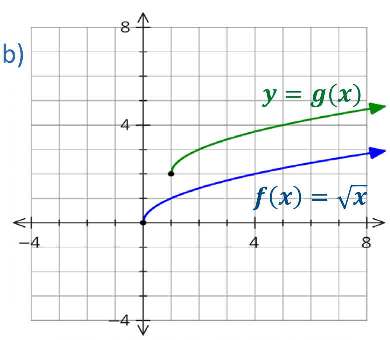
ii ________________

Equation of $g(x)$ in terms of x

ii ________________

Mapping rule of $y = f(x) \rightarrow y = g(x)$

(b)



i ________________

Equation of $g(x)$ in terms of $f(x)$

ii ________________

Equation of $g(x)$ in terms of x

ii ________________

Mapping rule of $y = f(x) \rightarrow y = g(x)$

Class Example 1.14 *Determining the Horizontal / Vertical Translation from the equation*

For each pair of functions below,

i - Describe how the graph of function ② can be obtained by transforming the graph of function ①.
ii – Provide a mapping rule for each.
iii – State the domain or range as prompted below

(a) ① $y = f(x)$
② $y = f(x+7) - 1$

i ______________________
Description of transformations from ① to ②

ii ______________________
Mapping rule

(b) ① $y = x^2$
② $y = (x-6)^2 + 4$

i ______________________
Description of transformations from ① to ②

ii ______________________
Mapping rule

iii ______________________
Range of function ②

(c) ① $f(x) = \sqrt{x+2}$
② $y = f(x-4) + 1$

i ______________________
Description of transformations from ① to ②

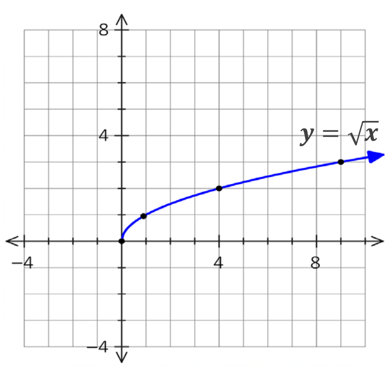
ii ______________________
Mapping rule

iii ______________________
Domain of function ②

Class Example 1.15 *Sketching a graph using translations*

Given each basic graph below, use transformations to **sketch** the indicated function on the same grid, and provide a mapping rule. Be sure to carefully transform each point indicated (•).
Indicate the domain and range of each sketched function. (Use either set or interval notation)

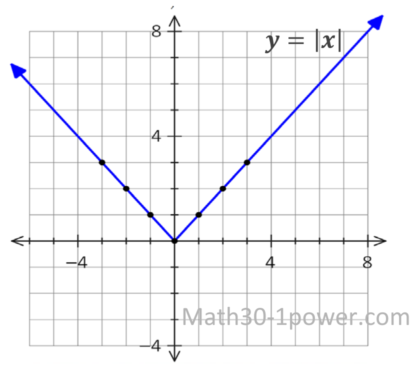
(a) Sketch $y = \sqrt{x+1} + 3$



____
Mapping rule

______________ ______________
Domain *Range*

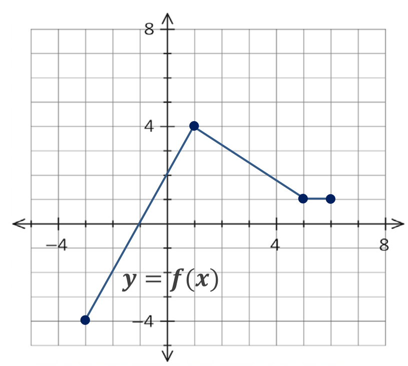
(b) Sketch $y = |x-2| + 4$



____
Mapping rule

______________ ______________
Domain *Range*

(c) Sketch $y = f(x-2) + 3$



____
Mapping rule

______________ ______________
Domain *Range*

Answers from previous page

1.13 (a) i $g(x) = f(x+2) + 3$
ii $(x, y) \rightarrow (x-2, y+3)$

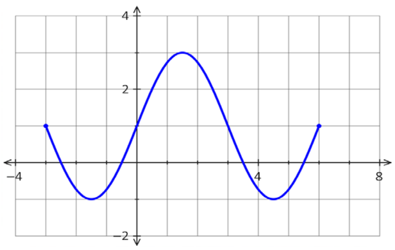
(b) i $g(x) = f(x+4)$
ii $g(x) = (x+4)^3$
iii $(x, y) \rightarrow (x-4, y)$

(c) i $g(x) = f(x-1) + 2$
ii $g(x) = \sqrt{x-1} + 2$
iii $(x, y) \rightarrow (x+1, y+2)$

1.1 Practice Questions

1. Given the graphs below, state the domain (D) and range (R) for each function, in the notations specified.

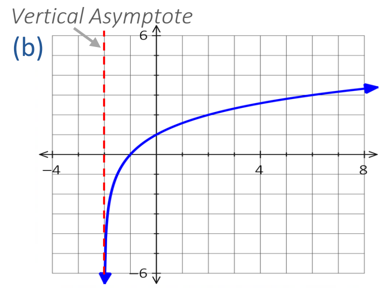
(a)



D: ________ *Set* ________ *Interval*

R: ________ *Set* ________ *Interval*

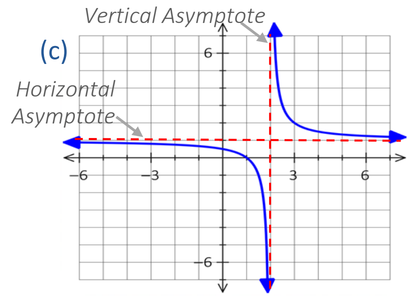
(b)



D: ________ *Set* ________ *Interval*

R: ________ *Set* ________ *Interval*

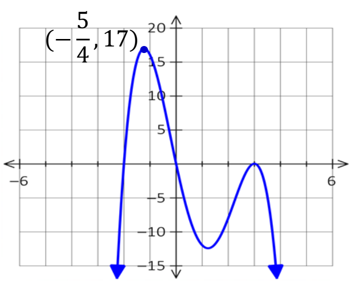
(c)



D: ________ *Set*

R: ________ *Set*

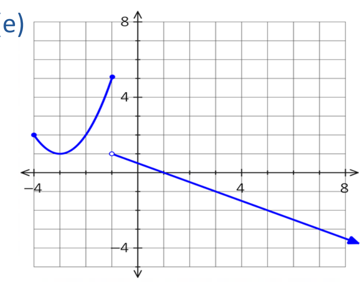
(d)



D: ________ *Set* ________ *Interval*

R: ________ *Set* ________ *Interval*

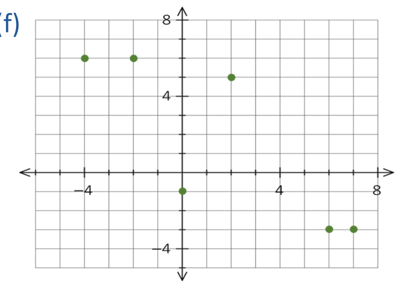
(e)



D: ________ *Set* ________ *Interval*

R: ________ *Set* ________ *Interval*

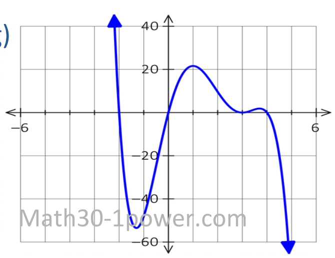
(f)



D: ________ *Set*

R: ________ *Set*

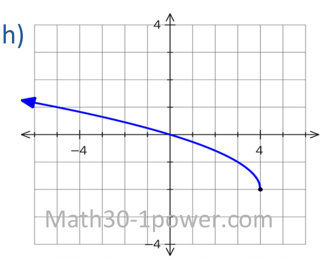
(g)



D: ________ *Set*

R: ________ *Set*

(h)



D: ________ *Interval*

R: ________ *Interval*

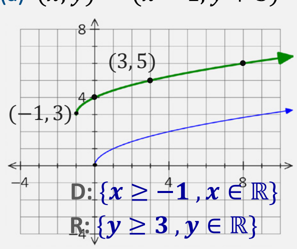
Answers from previous page

1.14 (a) i Horiz translation 7 units left, vert translation 1 unit down.
ii $(x, y) \rightarrow (x-7, y-1)$

(b) i Horiz translation 6 units right, vert translation 4 units up.
ii $(x, y) \rightarrow (x+6, y+4)$
iii $\{y \geq 4, y \in \mathbb{R}\}$ *or* $[4, \infty)$

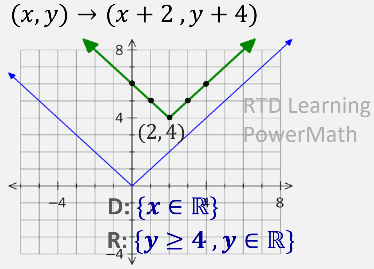
(c) i Horiz translation 4 units right, vert translation 1 unit up.
ii $(x, y) \rightarrow (x+4, y+1)$
iii $\{x \geq 2, x \in \mathbb{R}\}$ *or* $[2, \infty)$

1.15 (a) $(x, y) \rightarrow (x-1, y+3)$



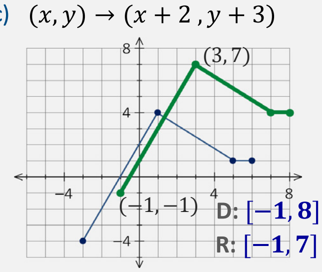
D: $\{x \geq -1, x \in \mathbb{R}\}$
R: $\{y \geq 3, y \in \mathbb{R}\}$

(b) $(x, y) \rightarrow (x+2, y+4)$



D: $\{x \in \mathbb{R}\}$
R: $\{y \geq 4, y \in \mathbb{R}\}$

(c) $(x, y) \rightarrow (x+2, y+3)$



D: $[-1, 8]$
R: $[-1, 7]$

2. Without graphing, state the domain and / or the range (as indicated) for each of the following functions.
Your choice – answer in either set or interval notation!

(a) $y = (x+3)^2 - 4$

D: ____________

R: ____________

(b) $h(x) = \sqrt{x+11} - 9$

D: ____________

R: ____________

(c) $y = \sqrt{6-9x} + 3$

D: ____________

R: ____________

(d) $f(x) = \dfrac{1}{x+3}$

D: ____________

(e) $p(x) = x^3 - 12x^2 + x - 4$

D: ____________

(f) $y = \dfrac{x+3}{x^2+x-6}$

D: ____________

(g) $g(x) = \dfrac{-8}{x^2+7}$

D: ____________

(h) $f(x) = 6$

D: ____________

R: ____________

(i) $y = -\sqrt{5x+1} - 11$

D: ____________

(j) $y = |5-2x| + 9$

D: ____________

R: ____________

(k) $y = 6x + 5$

D: ____________

(l) $g(x) = \dfrac{x^2-4}{x^2-3x-4}$

D: ____________

(m) $y = \dfrac{x}{x^2+x+5}$ D: ____________

(n) $y = 3^x + 1$ D: ____________

Answers to Practice Questions on the previous page

1. (a) D: $\{-3 \le x \le 6, x \in \mathbb{R}\}$ ***or*** $[-3, 6]$
R: $\{-1 \le y \le 3, y \in \mathbb{R}\}$ ***or*** $[-1, 3]$

(b) D: $\{x > -2, x \in \mathbb{R}\}$ ***or*** $(-2, \infty)$
R: $\{y \in \mathbb{R}\}$ ***or*** $(-\infty, \infty)$

(c) D: $\{x \neq 2, x \in \mathbb{R}\}$
R: $\{y \neq 1, y \in \mathbb{R}\}$

(d) D: $\{x \in \mathbb{R}\}$ ***or*** $(-\infty, \infty)$
R: $\{y \le 17, y \in \mathbb{R}\}$ ***or*** $(-\infty, 17]$

(e) D: $\{x \ge -4, x \in \mathbb{R}\}$ ***or*** $[-4, \infty)$
R: $\{y \le 5, y \in \mathbb{R}\}$ ***or*** $(-\infty, 5]$

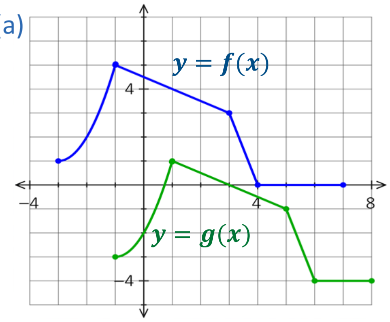
(f) D: $\{-4, -2, 0, 2, 6, 7\}$
R: $\{-3, -1, 5, 6\}$

(g) D: $\{x \in \mathbb{R}\}$ ***or*** $(-\infty, \infty)$
R: $\{y \in \mathbb{R}\}$ ***or*** $(-\infty, \infty)$

(h) D: $\{x \le 4, x \in \mathbb{R}\}$ ***or*** $(-\infty, 4]$
R: $\{y \ge -2, y \in \mathbb{R}\}$ ***or*** $[-2, \infty)$

3. For each pair of functions below, $y = g(x)$ is obtained by horizontally and / or vertically translating the graph of $y = f(x)$. Provide the indicated equations / mapping rule below.

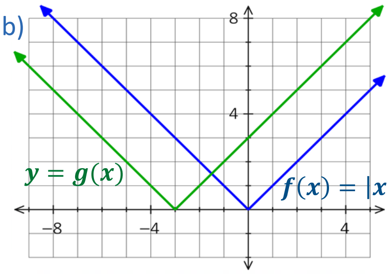
(a)



i ____________ *Equation of $g(x)$ in terms of $f(x)$*

ii ____________ *Mapping rule of $y = f(x) \rightarrow y = g(x)$*

(b)

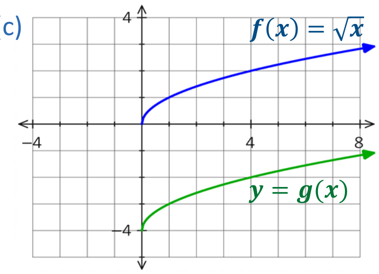


i ____________ *Equation of $g(x)$ in terms of $f(x)$*

ii ____________ *Equation of $g(x)$ in terms of x*

ii ____________ *Mapping rule of $y = f(x) \rightarrow y = g(x)$*

(c)



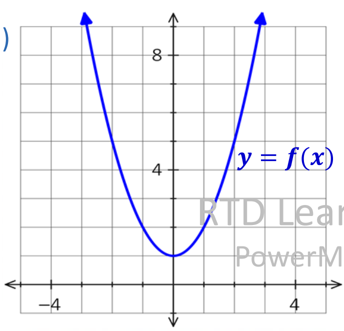
i ____________ *Equation of $g(x)$ in terms of $f(x)$*

ii ____________ *Equation of $g(x)$ in terms of x*

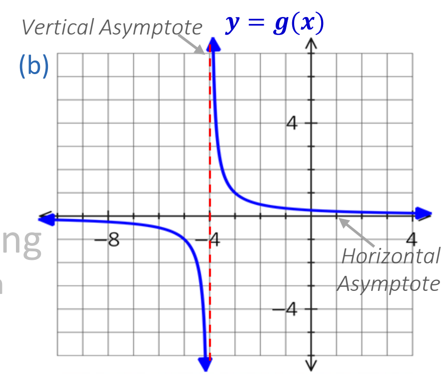
ii ____________ *Mapping rule of $y = f(x) \rightarrow y = g(x)$*

4. The graphs of each function below can be obtained by horizontally and / or vertically translating one of the basic graphs. Determine an equation for each function, in terms of x. Then, indicate the domain and range for each, in either set or interval notation.

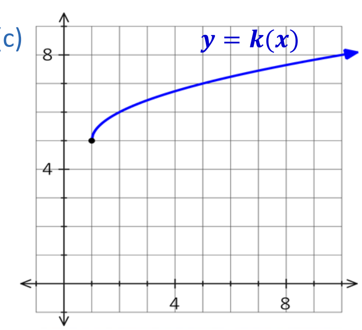
(a)



(b)



(c)



(a) i ____________ *Equation of $f(x)$*

ii ____________ *Domain* ____________ *Range*

(b) i ____________ *Equation of $g(x)$*

ii ____________ *Domain* ____________ *Range*

(c) i ____________ *Equation of $k(x)$*

ii ____________ *Domain* ____________ *Range*

Answers to Practice Questions on the previous page

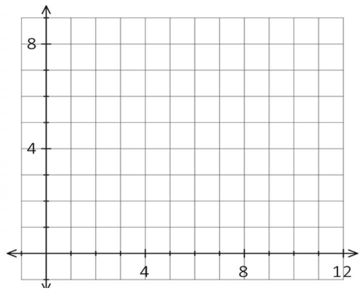
2. (a) D: $\{x \in \mathbb{R}\}$ *or* $(-\infty, \infty)$ R: $\{y \geq -4, y \in \mathbb{R}\}$ *or* $[-4, \infty)$
(b) D: $\{x \geq -11, x \in \mathbb{R}\}$ *or* $[-11, \infty)$ R: $\{y \geq -9, y \in \mathbb{R}\}$ *or* $[-9, \infty)$
(c) D: $\{x \leq 2/3, x \in \mathbb{R}\}$ *or* $(-\infty, \frac{2}{3}]$ R: $\{y \geq 3, y \in \mathbb{R}\}$ *or* $[3, \infty)$
(d) D: $\{x \neq -3, x \in \mathbb{R}\}$ (e) D: $\{x \in \mathbb{R}\}$ (f) D: $\{x \neq -3, 2, x \in \mathbb{R}\}$ (g) D: $\{x \in \mathbb{R}\}$ (h) D: $\{x \in \mathbb{R}\}$ R: $\{6\}$
(i) D: $\{x \geq -1/5, x \in \mathbb{R}\}$ (j) D: $\{x \in \mathbb{R}\}$ R: $\{y \geq 9, y \in \mathbb{R}\}$ (k) D: $\{x \in \mathbb{R}\}$ (l) D: $\{x \neq -1, 4,\ x \in \mathbb{R}\}$
(m) D: $\{x \in \mathbb{R}\}$ (n) D: $\{x \in \mathbb{R}\}$

5. The graphs of each function below can be obtained by horizontally and / or vertically translating one of the basic function graphs. Sketch each indicated function by plotting a minimum of 4 points. Then, indicate the domain and range of each sketched function, using either set or interval notation.

(a) $f(x) = \sqrt{x-4} + 3$

Domain

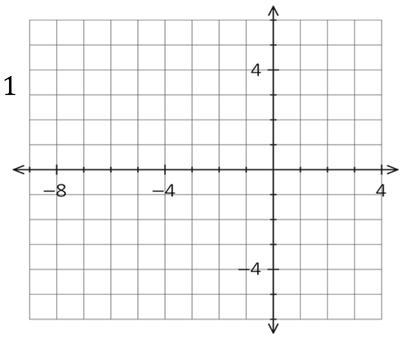
Range



(b) $g(x) = \dfrac{1}{x+4} + 1$

Domain

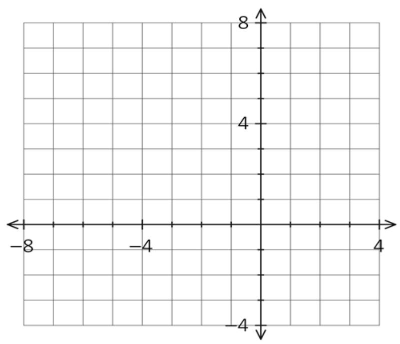
Range



(c) $h(x) = (x+5)^2 - 1$

Domain

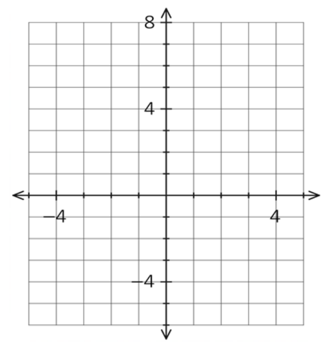
Range



(d) $y = x^3 + 2$

Domain

Range



6. For each pair of functions below,

i - Describe how the graph of function ② can be obtained by transforming the graph of function ①.
ii – Provide a mapping rule.
iii – State the domain or range where prompted below

(a) ① $y = 1/x$

② $y = \dfrac{1}{x-11} - 7$

i _Description of transformations from ① to ②_

ii _Mapping rule_

iii _Domain of function ②_

(b) ① $y = f(x)$

② $y = f(x+7) - 1$

i _Description of transformations from ① to ②_

ii _Mapping rule_

(c) ① $f(x) = (x-1)^2 + 2$

② $y = f(x+3) - 2$

i _Description of transformations from ① to ②_

ii _Mapping rule_

iii _Range of function ②_

Answers from previous page

3. (a) i $g(x) = f(x-2) - 4$ ii $(x, y) \to (x+2, y-4)$

(b) i $g(x) = f(x+3)$ ii $g(x) = |x+3|$ iii $(x, y) \to (x-3, y)$

(c) i $g(x) = f(x) - 4$ ii $g(x) = \sqrt{x} - 4$ iii $(x, y) \to (x, y-4)$

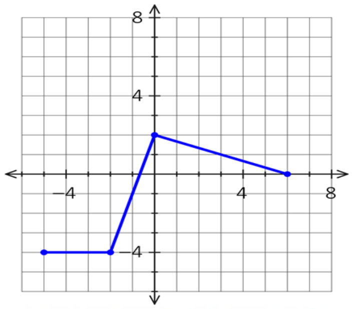
4. (a) i $f(x) = x^2 + 1$ ii _D:_ $\{x \in \mathbb{R}\}$ _R:_ $[1, \infty)$

(b) i $g(x) = \dfrac{1}{x+4}$ ii _D:_ $\{x \neq -4, x \in \mathbb{R}\}$ _R:_ $\{y \neq 0, y \in \mathbb{R}\}$

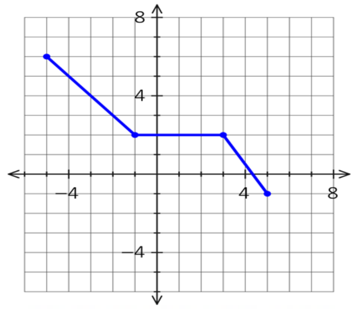
(c) i $k(x) = \sqrt{x-1} + 5$ ii _D:_ $\{x \geq 1, x \in \mathbb{R}\}$ _R:_ $\{y \geq 5, y \in \mathbb{R}\}$

7. Given each graph of the function $y = f(x)$, sketch the graph of the indicated function on the same grid.

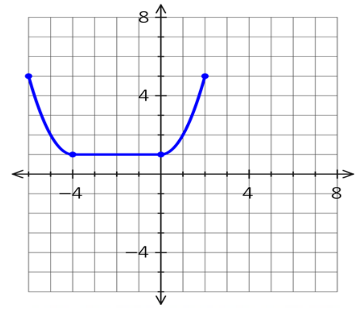
(a) $y = f(x) + 5$



(b) $y = f(x-2) - 4$



(c) $y = f(x-5) - 6$



8. Given each mapping rule for $y = f(x)$ to a transformed function, state an equation for the transformed function, in terms of $f(x)$.

(a) $(x, y) \to (x+1, y+2)$

(b) $(x, y) \to (x-5, y)$

9. The function $y = f(x)$ is transformed to $y + 4 = f(x-2)$. The point $P(-11, 5)$ is on the graph of $y = f(x)$.

(a) Describe the transformations from $y = f(x)$ to $y + 4 = f(x-2)$.

(b) Determine the new coordinates of the point P after the transformation.

10. A function $f(x) = x^2$ is transformed to $y = g(x)$ and $y = h(x)$ by applying vertical translations, with the effect described below. Determine the value and direction of the translation, and state an equation for the transformed function, in terms of $f(x)$ and in terms of x.

(a) Determine the vertical translation applied if $y = g(x)$ passes through $(3, 13)$.

i ______________

Translation

ii ______________

Equation of $g(x)$ in terms of $f(x)$

iii ______________

Equation of $g(x)$ in terms of x

(b) Determine the vertical translation applied if $y = h(x)$ passes through $(-4, 1)$.

i ______________

Translation

ii ______________

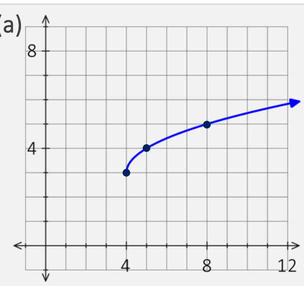
Equation in terms of $f(x)$

iii ______________

Equation in terms of x

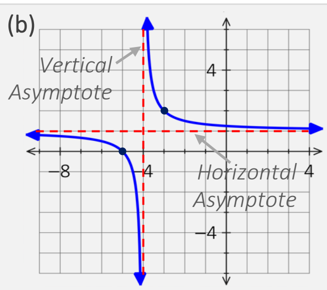
Answers to Practice Questions on the previous page

5. (a)



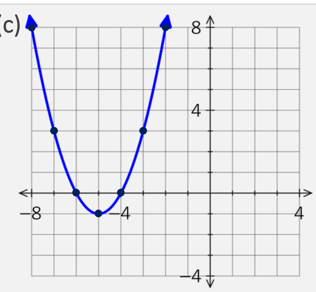
D: $[4, \infty)$ *R:* $[3, \infty)$

(b)



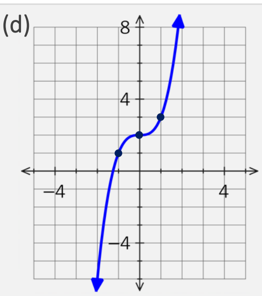
D: $\{x \neq -4, x \in \mathbb{R}\}$ *R:* $\{y \neq 1, y \in \mathbb{R}\}$

(c)



D: $(-\infty, \infty)$ *R:* $[-1, \infty)$

(d)



D: $\{x \in \mathbb{R}\}$ *R:* $\{y \in \mathbb{R}\}$

6. (a) i Horiz. translation 11 units right, vert. translation 7 units down
ii $(x, y) \to (x+11, y-7)$
iii *D:* $\{x \neq 11,\ x \in \mathbb{R}\}$

(b) i Horiz. translation 7 units left, vert. translation 1 unit down
ii $(x, y) \to (x-7, y-1)$

(c) i Horiz. translation 3 units left, vert. translation 2 units down
ii $(x, y) \to (x-3, y-2)$
iii *R:* $\{y \geq 0,\ y \in \mathbb{R}\}$

11. A function $f(x) = \sqrt{x}$ is transformed to $y = g(x)$ and $y = h(x)$ by applying horizontal translations, with the effect described below. Determine the value and direction of the translation, and state an equation for the transformed function, in terms of $f(x)$ and in terms of x.

(a) Determine the horizontal translation applied if $y = g(x)$ passes through $(7, 2)$.

i ____________ *Translation*

ii ____________ *Equation in terms of* $f(x)$

iii ____________ *Equation in terms of* x

(b) Determine the horizontal translation applied if $y = h(x)$ passes through $(-5, 5)$.

i ____________ *Translation*

ii ____________ *Equation in terms of* $f(x)$

iii ____________ *Equation in terms of* x

12. A function $f(x) = (x+2)^3 - 1$ is transformed to $y = g(x)$ by applying vertical translation, so that the graph of $g(x)$ passes through the point $(0, 2)$. Determine the value and direction of the translation, and state an equation for the transformed function, in terms of $f(x)$ and in terms of x.

i ____________ *Translation*

ii ____________ *Equation in terms of* $f(x)$

iii ____________ *Equation in terms of* x

13. A function $f(x) = \sqrt{x-1} + 4$ is transformed to $y = g(x)$ by applying horizontal translation 5 units to the left, and a vertical translation so that the graph of $g(x)$ passes through the point $(5, 15)$. Determine the value and direction of the vertical translation, and state an equation for the transformed function, in terms of $f(x)$ and in terms of x.

i ____________ *Translation*

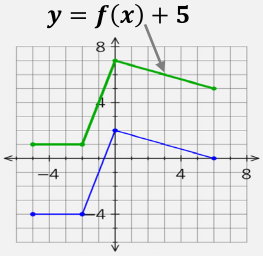
ii ____________ *Equation in terms of* $f(x)$

iii ____________ *Equation in terms of* x

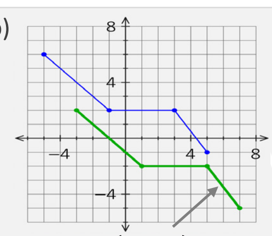
14. A function $g(x) = \dfrac{1}{x}$ is transformed to $y = k(x)$ by horizontally translating the graph 7 unit left and vertically translating 1 unit up, so the transformed graph passes through a point $P(-9, m)$. Determine the value of m.

Answers to Practice Questions on the previous page

7. (a) $y = f(x) + 5$

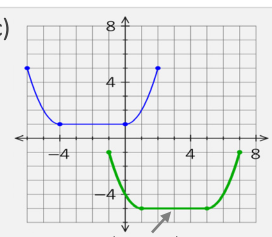


(b)



$y = f(x-2) - 4$

(c)



$y = f(x-5) - 6$

8. (a) $y = f(x-1) + 2$

(b) $y = f(x+5)$

9. (a) *Horiz translation 2 right, vertical translation 4 down.*

(b) ***P*** becomes $(-9, 1)$

10. (a) i vertical translation 4 units up

ii $g(x) = f(x) + 4$ iii $g(x) = x^2 + 4$

(b) i vertical translation 15 units down

ii $h(x) = f(x) - 15$ iii $h(x) = x^2 - 15$

15. The graph of a function $y = f(x)$, shown on the right, is transformed to create a function $g(x) - 9 = f(x + 5)$.

The point on the graph of $y = g(x)$ that corresponds to point P is in:

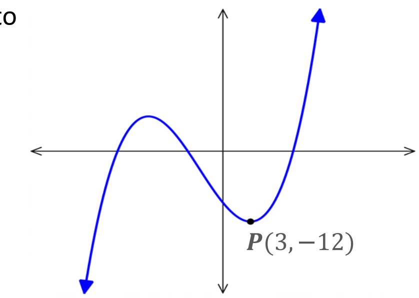
Exam Style

A. Quadrant I

B. Quadrant II

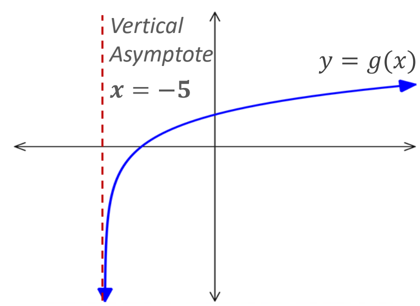
C. Quadrant III

D. Quadrant IV



16. The function $y = g(x)$, shown on the right, is transformed to $k(x) + 5 = g(x - 6)$.

State the domain and range of $y = k(x)$

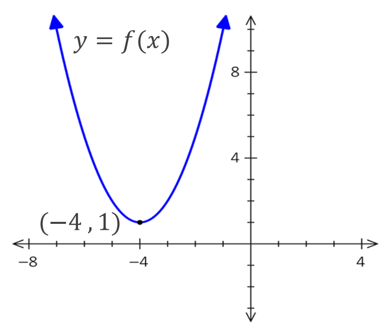


17. The graph of a function $y = f(x)$, shown on the right, has a vertex at $(-4, 1)$ and an equation that can be written in the form $y = (x - h)^2 + k$.
A new function $y = g(x)$ is defined $g(x) + 5 = f(x - 9)$.
Determine each of the following:

i ______
Domain of $g(x)$

ii ______
Range of $g(x)$

iii ______
Equation of $g(x)$ in terms of x



18. The function $y = f(x)$, shown on the right, is transformed to $y = g(x)$ by applying a horizontal translation.
If the graph of $y = g(x)$ passes through the point $(-2, 5)$, an equation for $y = g(x)$, in terms of $f(x)$, is:

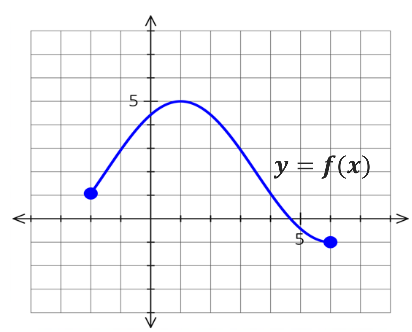
Exam Style

A. $y = f(x + 4)$

B. $y = f(x - 4)$

C. $y = f(x - 3)$

D. $y = f(x + 3)$



Answers to Practice Questions on the previous page ***and this page***

11. (a) i horizontal translation 3 units right
ii $g(x) = f(x - 3)$ iii $g(x) = \sqrt{x - 3}$
(b) i horizontal translation 30 units left
ii $g(x) = f(x + 30)$ iii $g(x) = \sqrt{x + 30}$

12. i vertical translation 5 units down ii $g(x) = f(x) - 5$ iii $g(x) = (x + 2)^3 - 6$

13. i vertical translation 8 units up ii $g(x) = f(x) + 8$ iii $g(x) = \sqrt{x + 4} + 12$ **14.** $m = 1/2$

15. C **16.** D: $\{x > 1, x \in \mathbb{R}\}$ R: $\{x \in \mathbb{R}\}$ **17.** i $\{x \in \mathbb{R}\}$ ii $\{y \geq -4, y \in \mathbb{R}\}$ iii $y = (x - 5)^2 - 4$ **18.** D